Beyond Getting it Right! Fostering Math Understanding in the Classroom

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What the Researchers Say

“There are five attributes associated with the concept of proficiency: 1) conceptual understanding (comprehension of mathematical concepts, operations, and relations), 2) procedural fluency (skills in carrying out procedures flexibly, fluently, and appropriately), 3) strategic competence (ability to formulate, represent, and solve mathematical problems), 4) adaptive reasoning (capacity for logical thought, reflection, explanation, and justification), and 5) productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).”

Adding It Up, Report of the National Research Council, 2001

“Proficient students expect mathematics to make sense. They take an active stance in solving mathematical problems. When faced with a non-routine problem, they have the courage to plunge in and try something, and they have the procedural and conceptual tools to carry through. They are experimenters and inventors, and can adapt known strategies to new problems. They think strategically.”

National Mathematics Standards (Draft), 2009

“To prepare students for Algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency, and problem-solving skills. Debates regarding the relative importance of these aspects of mathematical knowledge are misguided. These capabilities are mutually supportive, each facilitating learning of the others; taken together, conceptual understanding of mathematical operations, fluent execution of procedures, and fast access to number combinations jointly support effective and efficient problem solving.”


“Students who care about being precise, who look for hidden structure and note regularity in repeated reasoning, who make sense of complex problems and persevere in solving them, who construct viable arguments and use technology intelligently are more likely to be able to apply the knowledge they have attained...”

National Mathematics Standards (Draft), 2009
What the Researchers Say

“...flexibility in the use of mathematical procedures is paramount in algebra, given that procedures play a particularly prominent role in mathematics. In the domain of algebra, flexibility includes the ability to select appropriate procedures for particular problems and modify and adapt procedures to fit the conditions.”

Adding It Up: Helping Children Learn Mathematics
National Research Council 2001

“Both international and national mathematics assessments indicate that, while U.S. students may learn to execute mathematical procedures they often fail to gain the kind of robust, flexible knowledge that would allow them to apply what they learned to new situations and novel problems.”

Using Contrasting Examples to Support Procedural Flexibility and Conceptual Understanding in Mathematics
Rittle-Johnson & Star 2005

“Comparison is a powerful tool that mathematics teachers can use to introduce students to multiple strategies. Comparison can and should play a key role in the teaching of algebra...”

It Pays to Compare: Using Comparison to Help Build Student's Flexibility in Mathematics
Star 2008

A Taxonomy of Constructive Uses of Errors

Adapted from Students' Constructive Uses of Mathematical Errors: A Taxonomy (Borasi 1989)

<table>
<thead>
<tr>
<th></th>
<th>Performing a Mathematical Task</th>
<th>Understanding Mathematical Content</th>
<th>Understanding about Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Remediation</strong></td>
<td>Analyze recognized errors to understand what went wrong and fix it</td>
<td>Analyze recognized errors so as to clarify content and concept misunderstandings</td>
<td>Analyze recognized errors so as to clarify misunderstandings about the nature or process of mathematics</td>
</tr>
<tr>
<td><strong>Conceptual Learning</strong></td>
<td>Use errors to prevent and eliminate missteps in the process of learning new tasks</td>
<td>Use errors to probe and refine meaning in the process of learning new concepts</td>
<td>Use errors to increase awareness of the process and form of mathematical thinking</td>
</tr>
<tr>
<td><strong>Exploration</strong></td>
<td>Use errors to motivate questions that lead to new mathematical directions and tasks</td>
<td>Use errors to motivate questions that may provide new perspectives or insights into concepts</td>
<td>Use errors to motivate questions and insights into the nature of mathematical thinking</td>
</tr>
</tbody>
</table>
**Possible Roles on Teams**

Student teams often function most effectively when members have designated roles. These can be instructor-determined or established by the groups themselves, e.g., by giving teams a list such as the one below and asking them to decide on and delegate appropriate roles within their group.

The roles you – or your students assign will depend on the goals of the assignment, the size of the team, etc. They can be fixed or rotating. Here are some possible group roles, but the list is not exhaustive. Think creatively and come up with your own!

**Facilitator**: Moderates team discussion, keeps the group on task, and distributes work.

**Recorder**: Takes notes summarizing team discussions and decisions, and keeps all necessary records.

**Reporter**: Serves as group spokesperson to the class or instructor, summarizing the group’s activities and/or conclusions.

**Timekeeper**: Keeps the group aware of time constraints and deadlines and makes sure meetings start on time.

**Devil’s Advocate**: Raises counter-arguments and (constructive) objections, introduces alternative explanations and solutions.

**Harmonizer**: Strives to create a harmonious and positive team atmosphere and reach consensus (while allowing a full expression of ideas).

**Prioritizer**: Makes sure group focuses on most important issues and does not get caught up in details.

**Explorer**: Seeks to uncover new potential in situations and people (fellow team members but also clients) and explore new areas of inquiry.

**Innovator**: Encourages imagination and contributes new and alternative perspectives and ideas.

**Checker**: Checks to make sure all group members understand the concepts and the group’s conclusions.

**Runner**: Gets needed materials and is the liaison between groups and between their group and the instructor.

**Wildcard**: Assumes the role of any missing member and fills in wherever needed.

These roles are adapted from lists in:


Role Card #1
Resource Manager

- Make sure that the team has all materials or resources for the tasks.
- Make sure your team cleans up by delegating tasks. You could say, "I will put away the _____ while you ________.
- Check for accuracy and clarity of thinking during discussion. May also check written work and keeps track of team point scores.

Role Card #2
Facilitator

- Help the team agree on an answer and lead discussion when there are disagreements.
- Make sure you start the discussion in your group to maximize learning. You could say: "What do we think about the problem?" "Do we agree on all parts of the solution?"

Role Card #3
Recorder/Reporter

- Make sure you keep notes on important thoughts expressed in the group.
- Make final summary and lead the presentation while delegating other tasks needed for reporting results or your group work.
- Take notes for the team and notes should include phrases like, “For part one (or step one) ...” and explanations like, “Because part one (or step one) is ...”

Role Card #4
Task Manager

- Remind the team to stay on task and not to talk to students in other teams. You can suggest “Let us move on to the next step because ... “
- Listen for reasons and challenge your teammates to justify their thinking. “Why do you notice..." or “Why do you think that?"
- Keep track of time and remind team how much time is left to complete assigned task.
Play the Role

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Use Play the Role to give each student in the group an opportunity to explore and gain a deeper understanding of an article, mathematical task, text, etc.</th>
</tr>
</thead>
</table>
| Process | 1. Label 4 note cards with the group roles and their definitions:  
- **Summarizer**: highlights key ideas in the article or mathematical task  
- **Questioner**: poses questions to clarify ambiguous concepts  
- **Clarifier**: addresses confusing part of the article or mathematical task and attempts to answer the questions posed by the Questioner  
- **Predictor**: makes guesses or assumptions about where the article is going next or possible answers there could be even before arriving at an answer of a mathematical task  
  
2. Have students get into groups of 4.  
3. Give each student a note card that identifies his/her role in the group.  
4. Have students read a few paragraphs from the article or a mathematical task. Ask them to take notes on post-its as they read to prepare for their roles in the discussion.  
5. Once they have finished reading the paragraphs from the article, have students play their roles in the discussion.  
6. The roles in the group then switch one person to the right and students do the next mathematical task.  
7. Students repeat the process using new roles. This process continues until students play all the roles. |
### Four A Questions

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Use the Four A Questions protocol (to do a close reading of an article(s)) to generate discussion among students.</th>
</tr>
</thead>
</table>
| Process | 1. Post the Four A Questions:  
- What **Assumptions** does the author of the article hold?  
  What **Assumptions** do you team have about the mathematical task?  
- What do you **Agree** with in the article?  
  What do you **Agree** with in the answer or approach your classmate has?  
- What do you want to **Argue** with in the text?  
  What do you want to **Argue** with in the answer or approach your classmate presented?  
- What parts of the article do you **Aspire** to?  
  What does your team **Aspire** to come up with to make sense of the problem? |
|         | 2. Have students read the article or read and do the mathematical task and take notes in response to the Four A questions. |
|         | 3. Ask students to get into groups of 4. Have each group member take one of the Four A questions. |
|         | 4. Starting with the Assumption question, ask students to answer their questions. Then have group members discuss the answers given and respond to each other. |
|         | 5. Once each group has talked about all four questions, bring the whole group together for a discussion of the Four A questions. |

*Adapted from National School Reform Faculty  
Compiled by Jobs for the Future, 2008*
# Chalk Talk

<table>
<thead>
<tr>
<th><strong>Purpose</strong></th>
<th>Use Chalk Talk as a way to promote discussion and awareness of issues and perspectives – silently. It is also a way to promote awareness of patterns and problems and to ensure that all voices are heard.</th>
</tr>
</thead>
</table>
| **Process** | 1. Formulate an important, open-ended question or problem that will provoke comments, responses and multiple strategies to solve it.  
2. Provide plenty of chart paper and colored pencils and/or colored post-it notes and arrange a good space for students to write and respond. Write the question or problem in the middle of the paper in bold marker.  
3. Explain the chalk talk protocol and answer any student questions.  
4. Set-up norm for the chalk talk: This technique only works if everyone is writing and responding throughout the designated time period. Make it clear that everyone is responsible for writing, reading other people’s comments, and responding; there should be no talking; and no one should sit down until the time period is over. Opinions must be freely expressed and honored, and no personal attacks are allowed.  
5. Allow 10 – 20 minutes for the chalk talk. As facilitator, it’s helpful to walk around and read, and gently point students to interesting comments. All writing and responding is done in silence.  
6. Search for patterns. In pairs, students should read through all the postings and search for patterns and themes (or “notice and wonder”). This part takes about 5 minutes.  
7. Whole group share: Pairs should report out patterns and themes, round-robin style, until all perceptions are shared.  
8. Process debrief: What was the experience like of “talking” silently? |

*Adapted from IRA/NCTE, 2014*  
Compiled by Expeditionary Learning
### Purpose
Use the 1 – 3 – 6 protocol to create a forum for students to develop their own ideas and opinions about an article(s) or mathematical task(s), to share their ideas and opinions with a group and build on other’s ideas and opinions during discussion.

### Process
1. Give students an article or articles to read or a mathematical task(s) to solve.

2. Have students write their responses to the article(s) or write their answer(s) to a mathematical task(s).

3. Then have students move into groups of three where they share their ideas or mathematical thinking with the group. Have each group cluster their ideas together and write a list of them on a chart paper. In mathematics, have students chart their different approaches or strategies to analyze and solve the problem.

4. Join groups of three together to make groups of six. Again, have students share their ideas in groups of six.

5. Have the groups of six write a list of their ideas and bring them together and clarify the lists of ideas from the group of three.

6. Ask each group of six to share their list with the whole group.

**Materials**
Chart papers, Markers

*Adapted from Joanne McCabe*
Compiled by Jobs for the Future, 2008
1-3-6

READ

WRITE

SHARE

LIST

SHARE

LIST

REPORT OUT

Adapted from Collaborative Group Protocols, Jobs for the Future, 2008, p. 4-5
# 3 – 2 – 1

<table>
<thead>
<tr>
<th><strong>Purpose</strong></th>
<th>Use the 3 – 2 – 1 protocol to focus on discussion of an article or a mathematical task to encourage students to join the discussion and share their learning.</th>
</tr>
</thead>
</table>
| **Process** | 1. Post the 3 – 2 – 1 questions:  
   - What are 3 things you learned?  
   - What are 2 things you found interesting?  
   - What is 1 question you still have?  
   
   2. Have students read the article(s) or mathematical task(s) and take notes in response to the 3 – 2 – 1 questions.  
   
   3. Ask students to get into groups of 3 – 4 people to discuss their responses to the questions.  
   
   4. Once students have answered all of the questions, have them create a visual to share with the group that represents their answers to the questions.  
   
   5. Bring the whole group together and have each small group share their visual.  
   
   6. Discuss the article(s) or mathematical task(s) and the answers to the 3 – 2 – 1 questions.  
   
   This can also be used as an exit ticket to gauge whether students were able to meet the learning goal(s) of the lesson. |

*Materials*

Chart papers, Markers

*Adapted from Lipton and Wellman*

Compiled by Jobs for the Future, 2008
# Sentence Starters

<table>
<thead>
<tr>
<th><strong>Predicting</strong></th>
<th><strong>Expressing an Opinion</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><em>I guess/predict/imagine that</em></td>
<td></td>
</tr>
<tr>
<td><em>Based on______, I infer that...</em></td>
<td></td>
</tr>
<tr>
<td><em>I hypothesize that ___ because...</em></td>
<td></td>
</tr>
<tr>
<td><strong>Asking for Clarification</strong></td>
<td><strong>Paraphrasing</strong></td>
</tr>
<tr>
<td><em>What do you mean?</em></td>
<td></td>
</tr>
<tr>
<td><em>Will you explain that again?</em></td>
<td></td>
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<tr>
<td><em>Would explain how you found your answer?</em></td>
<td></td>
</tr>
<tr>
<td><strong>Soliciting a Response</strong></td>
<td><strong>Acknowledging Ideas</strong></td>
</tr>
<tr>
<td><em>What do you think?</em></td>
<td></td>
</tr>
<tr>
<td>*We haven’t heard from you yet. *</td>
<td></td>
</tr>
<tr>
<td><em>Do you agree?</em></td>
<td></td>
</tr>
<tr>
<td><em>My ideas is similar to/related to <em><strong>’s idea in that</strong></em>_____</em></td>
<td></td>
</tr>
<tr>
<td><em>I agree with ___ that____ because______.</em></td>
<td></td>
</tr>
<tr>
<td><em>My idea builds upon ___’s idea.</em></td>
<td></td>
</tr>
<tr>
<td><strong>Reporting a Partner’s Idea</strong></td>
<td><strong>Reporting a Group’s Idea</strong></td>
</tr>
<tr>
<td><em>___ shared with me that...</em></td>
<td></td>
</tr>
<tr>
<td><em>___ pointed that out...</em></td>
<td></td>
</tr>
<tr>
<td><em>___ concluded that...</em></td>
<td></td>
</tr>
<tr>
<td><em>We decided/agreed that...</em></td>
<td></td>
</tr>
<tr>
<td><em>We concluded that...</em></td>
<td></td>
</tr>
<tr>
<td><em>Our group sees it differently.</em></td>
<td></td>
</tr>
<tr>
<td><em>We had a different approach.</em></td>
<td></td>
</tr>
<tr>
<td><strong>Disagreeing</strong></td>
<td><strong>Offering a Suggestion</strong></td>
</tr>
<tr>
<td><em>I don’t agree with you because...</em></td>
<td></td>
</tr>
<tr>
<td>*I got a different answer than you. *</td>
<td></td>
</tr>
<tr>
<td><em>I see it another way.</em></td>
<td></td>
</tr>
<tr>
<td><em>Maybe we could...</em></td>
<td></td>
</tr>
<tr>
<td><em>What if...</em></td>
<td></td>
</tr>
<tr>
<td><em>If we thought about it in a different way, we might see that...</em></td>
<td></td>
</tr>
<tr>
<td><strong>Affirming</strong></td>
<td><strong>Holding the Floor</strong></td>
</tr>
<tr>
<td>*That’s an interesting idea. *</td>
<td></td>
</tr>
<tr>
<td>*I hadn’t thought of that. *</td>
<td></td>
</tr>
<tr>
<td><em>I see what you mean.</em></td>
<td></td>
</tr>
<tr>
<td><em>As I was saying...</em></td>
<td></td>
</tr>
<tr>
<td><em>If I could finish my thought...</em></td>
<td></td>
</tr>
<tr>
<td><em>What I was trying to say was...</em></td>
<td></td>
</tr>
</tbody>
</table>

Adapted from Dr. Kate Kinsella
# Developing Mathematical Thinking with Effective Questions

<table>
<thead>
<tr>
<th>To help students build confidence and rely on their own understanding, ask...</th>
<th>To help students collectively make sense of mathematics, ask...</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ Does that make sense?</td>
<td>✓ What do you think about what __said?</td>
</tr>
<tr>
<td>✓ Why is that true?</td>
<td>✓ Do you agree? Why or why not?</td>
</tr>
<tr>
<td>✓ Can you make a model to show that?</td>
<td>✓ Does anyone have the same answer but a different way to explain it?</td>
</tr>
<tr>
<td>✓ How did you reach that conclusion?</td>
<td>✓ Can you convince the rest of us that your answer makes sense?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>To help students learn to reason mathematically, ask...</th>
<th>To encourage conjecturing, ask...</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ Is that for all cases? Explain.</td>
<td>✓ What would happen if ...? What if not?</td>
</tr>
<tr>
<td>✓ Can you think of a counterexample?</td>
<td>✓ Do you see a pattern? Can you explain the pattern?</td>
</tr>
<tr>
<td>✓ How would you prove that?</td>
<td>✓ What are some possibilities here?</td>
</tr>
<tr>
<td>✓ What assumptions are you making?</td>
<td>✓ Can you predict the next step? What about the last?</td>
</tr>
<tr>
<td>✓ Can you cite evidence to justify your thinking?</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>To promote problem solving, ask...</th>
<th>To make connections among ideas and applications, ask...</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ What do you need to find out?</td>
<td>✓ How does this relate to...?</td>
</tr>
<tr>
<td>✓ What information do you have?</td>
<td>✓ What ideas that we have learned before were useful in solving this problem?</td>
</tr>
<tr>
<td>✓ What strategies are you going to use?</td>
<td>✓ Can you give me an example of ...?</td>
</tr>
<tr>
<td>✓ What tools will you need? Calculator? Tape diagram? Number discs?</td>
<td>✓ Do you see a similar structure from previous lessons that can be applied to this problem?</td>
</tr>
<tr>
<td>✓ What do you think the answer or result will be?</td>
<td></td>
</tr>
<tr>
<td>To help when students get stuck, ask...</td>
<td>To check student progress, ask...</td>
</tr>
<tr>
<td>---------------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>✓ How would you describe the problem in your own words?</td>
<td>✓ Can you explain what you have done so far? What else is there to do?</td>
</tr>
<tr>
<td>✓ What do you know that is not stated in the problem?</td>
<td>✓ Why did you decide to use this method?</td>
</tr>
<tr>
<td>✓ What facts do you have?</td>
<td>✓ Can you think of another method that might have worked?</td>
</tr>
<tr>
<td>✓ Could you try it with simpler number? Fewer numbers? Using a number line?</td>
<td>✓ Is there a more efficient strategy?</td>
</tr>
<tr>
<td>✓ What about putting things in order?</td>
<td>✓ What do you notice when...?</td>
</tr>
<tr>
<td>✓ How did you tackle similar problems?</td>
<td>✓ Why did you decide to organize your results like that?</td>
</tr>
<tr>
<td>✓ Would it help to create a diagram? Make a table? Draw a picture?</td>
<td>✓ Do you think this would work with other numbers?</td>
</tr>
<tr>
<td>✓ Can you guess and check?</td>
<td>✓ Have you thought of all the possibilities? How can you be sure?</td>
</tr>
<tr>
<td>✓ Have you compared your work with anyone else? What did other members of your group try?</td>
<td>✓ Explain what you have done so far. What else is there to do?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>To encourage reflection, ask...</th>
<th>What other questions would you like to add to this list?</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ How did you get your answer?</td>
<td>✓ What other questions would you like to add to this list?</td>
</tr>
<tr>
<td>✓ Does your answer seem reasonable? Why or why not?</td>
<td></td>
</tr>
<tr>
<td>✓ Can you describe your method to us all? Can you explain why it works?</td>
<td></td>
</tr>
<tr>
<td>✓ What if you had started with... rather than...?</td>
<td></td>
</tr>
<tr>
<td>✓ What if you could only use...?</td>
<td></td>
</tr>
<tr>
<td>✓ What have you learned or found out today?</td>
<td></td>
</tr>
<tr>
<td>✓ Did you use or learn any new words today? What do they mean? How do you spell them?</td>
<td></td>
</tr>
<tr>
<td>✓ What are the key points or big ideas in this lesson?</td>
<td></td>
</tr>
</tbody>
</table>

Adapted from PBS TeacherLine
## Developing Mathematical and Statistical Questions

**The Parking Spot**

![Image](image.jpg)

### Part 1:

<table>
<thead>
<tr>
<th>What is the first question that comes to your mind upon seeing the picture?</th>
</tr>
</thead>
<tbody>
<tr>
<td>What mathematical question can you ask about the picture?</td>
</tr>
<tr>
<td>What is a possible answer to your mathematical question?</td>
</tr>
<tr>
<td>What facts, information or variables do you need to know to answer the mathematical question?</td>
</tr>
<tr>
<td>How will you approach the problem to answer the mathematical question? What strategies could you use to answer it?</td>
</tr>
</tbody>
</table>
### Part 2:

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is the first question that comes to your mind upon seeing the picture?</td>
<td></td>
</tr>
<tr>
<td>What statistical question can you ask about the picture?</td>
<td></td>
</tr>
<tr>
<td>What facts, information or data do you need to gather to answer the statistical question?</td>
<td></td>
</tr>
<tr>
<td>How do you gather the facts, information or data? What sampling method would you use? Why?</td>
<td></td>
</tr>
<tr>
<td>What do you want to measure or find from the data gathered?</td>
<td></td>
</tr>
<tr>
<td>How would you know if the result is statistically significant? What measure would you use? Why?</td>
<td></td>
</tr>
</tbody>
</table>

*Developed by Bernadette A. Salgarino, Ed.D.*
Vocabulary Development

Put It In Context!

- Review the list of vocabulary words and the figure shown below.
- Record the vocabulary words from the list that are represented in the figure. You may add words not in the list.
- Explain/discuss where each highlighted word is represented in the figure.
- Explain/discuss why some of the words were not included.
- Explain/discuss how knowing the meaning of each selected word would help you understand the concept.

Vocabulary List

<table>
<thead>
<tr>
<th>Pre-image</th>
<th>Rotation</th>
<th>Congruent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image</td>
<td>Reflection</td>
<td>Equidistant</td>
</tr>
<tr>
<td>Bisector</td>
<td>Isometry</td>
<td>Vector</td>
</tr>
<tr>
<td>Parallel</td>
<td>Translation</td>
<td>Collinear</td>
</tr>
<tr>
<td>Symmetry</td>
<td>Perpendicular</td>
<td>Fixed Points</td>
</tr>
</tbody>
</table>
Put It In Context!

Vocabulary List

Pre-image | Rotation | Congruent
---|---|---
Image | Reflection | Equidistant
Bisector | Isometry | Vector
Parallel | Translation | Collinear
Symmetry | Perpendicular | Fixed Points
Sample Mathematical Tasks

Grade 6

Cargo Truck

Cube-shaped boxes will be loaded into the cargo hold of a truck. The cargo hold of the truck is in the shape of a rectangular prism. The edges of each box measure 2.50 feet and the dimensions of the cargo hold are 7.50 feet by 15.00 feet by 7.50 feet, as shown below.

What is the volume, in cubic feet, of each box?

Determine the number of boxes that will completely fill the cargo hold of the truck. Use words and/or numbers to show how you determined your answer.
Four identical right triangles are arranged inside a rectangle as shown. The figure is **not** drawn to scale. What is the area of one of the right triangles?

Answer: _______________________

Explain how you arrived at your answer.
**Round Trip Walk**

_Solve the problem below. Show your work in the box._

The graph shown below represents a person’s round trip walk to a store starting from home. Which of the following best describes the 45 minute trip?

A. The person walked uphill, then on a flat surface, then downhill.
B. The person walked toward the store, stopped, then walked back home.
C. The person walked uphill, stopped, then walked downhill back home.

Answer: ______

---

Explain what helped you make your decision.

*Adapted from the Milwaukee Mathematics Partnership*
• Experience a Smarter Balanced performance task from the perspective of a student.
• Understand alignment with Smarter Balanced Claims and CCSS Standards for Mathematical Practice.
• Understand and use Scoring Guides to score student responses to hand-scored items within the task.
Agenda

• Welcome and Introduction
• Challenge
• Grade 7 Assessment Literacy Toolkit
• Grade 8 Assessment Literacy Toolkit
• Questions and Reflections
Lenses to Consider in this Session

Learner Lens  Teacher Lens
Common Core State Standards

As the CA CCSS-M are implemented, students will face increased language demands during mathematics instruction. Students will be asked to engage in discussions on mathematics topics, to explain their reasoning, to demonstrate their understanding, and to listen to and critique the reasoning of others. The increased language demands may pose challenges for all students and even greater challenges for English learners.

-CA Mathematics Framework Universal Access Chapter Page 9
CCSS Mathematical Practices

OVERARCHING HABITS OF MIND
1. Make sense of problems and persevere in solving them
6. Attend to precision

REASONING AND EXPLAINING
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others

MODELING AND USING TOOLS
4. Model with mathematics
5. Use appropriate tools strategically

SEEING STRUCTURE AND GENERALIZING
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning
Why Performance Task?

Integrating knowledge and skills across multiple content standards...
Smarter Balanced Assessment Consortium: Performance Task Specifications

Role of Smarter Balanced Performance Tasks

Taken during the final 12 weeks of the school year, the Smarter Balanced summative assessments for accountability will have two components: a comprehensive end-of-year computer adaptive assessment and performance tasks. These assessments in English language arts (ELA) and mathematics will provide measures of students’ achievement (proficiency in meeting grade-level standards), academic growth, and progress toward college and career readiness. The focus of both assessment components will be the claims and targets identified in the Smarter Balanced content specifications for ELA/literacy and mathematics, which serve as “bridge documents” between the Common Core State Standards and the Smarter Balanced summative assessments. Through the use of technology and innovative item and task formats, the Smarter Balanced assessments will exemplify “next generation assessments,” significantly improving upon traditional, large-scale accountability tests in terms of authenticity, accessibility, and coverage of skills that are identified in college and career standards (e.g., mathematics practices, problem solving, speaking and listening, use of technology), as described in the Smarter Balanced content specifications.

The domain of performance assessment is quite broad, encompassing a range of non-selected-response tasks. A Smarter Balanced performance task involves significant interaction of students with stimulus materials and/or engagement in a problem solution, ultimately leading to an exhibition of the students’ application of knowledge and skills, often in writing or spoken language. Stimuli include a variety of information forms (e.g., readings, video clips, data), as well as an assignment or problem situation. A key component of college and career readiness is the ability to integrate knowledge and skills across multiple content standards. Smarter Balanced will address this ability through performance tasks, because it cannot be adequately assessed with selected-response or constructed-response items.
Key Phrases

- **Interaction** with varied, rich stimuli
- **Engages students** in a scenario
  - Solve a problem
  - Create a product with a specific purpose
- **Application** of knowledge and skills
- **Integration** . . . across multiple standards
- **Assesses** what selected- and constructed-response items cannot
Reflect . . .

Based on your analysis of the “Role of Smarter Balanced Performance Tasks” document, why is Smarter Balanced using performance tasks in its summative assessments?
The Short Answer . . .

College and Career Readiness
The Role of UDL in Smarter Balanced Summative Assessments and Our Own Classroom Instruction

Universal Design for Learning Guidelines

Provide Multiple Means of Engagement
Purposeful, motivated learners

Provide Multiple Means of Representation
Resourceful, knowledgeable learners

Provide Multiple Means of Action & Expression
Strategic, goal-directed learners
Structure of a Smarter Balanced Performance Task

• Each performance task has an explicit overall task stated within the Stimulus.
• Each performance task has a total of six questions (items), some with multiple components.
• First two questions are to support understanding of the context of the Stimulus, and are typically machine-scored.
• Questions 3–6 build to resolve an overarching question, and some or all of these are hand-scored.
• Interdependencies exist among questions 3–6 in a controlled manner.
• Each question within a performance task is aligned to the CCSS, the SMPs, DOK, and Smarter Balanced Claims.
ASSESSMENT CONTINUUM

Knowledge
Master Core Academic Content

Knowledge & Skills
Master Core Academic Content
Think Critically
Communicate Effectively
Solve Complex Problems

Knowledge, Skills, Work Habits & Dispositions
Master Core Academic Content
Think Critically
Communicate Effectively
Solve Complex Problems
Work Collaboratively
Learn How to Learn

Depth of Learning

ON-DEMAND, STANDARDIZED ITEMS AND TASKS
Selected Response Items
Short Constructed Response Items
Long Constructed Response Items
Standardized Performance Tasks

CURRICULUM-EMBEDDED PERFORMANCE TASKS
Complex Performance Tasks
Complex Extended Projects

(c) 2011 Stanford Center for Assessment, Learning, & Equity
Smarter Balanced Item Types

<table>
<thead>
<tr>
<th>CAT Assessment Items</th>
<th>Performance Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>– Focus on grade-level content skills</td>
<td>– Focus on students’ ability to problem solve in real-life situations</td>
</tr>
<tr>
<td>– Computer adaptive (item difficulty depends on response to each prior item)</td>
<td>– Focus on previous grade-level content skills, with some integration of on-grade-level skills</td>
</tr>
<tr>
<td>– Questions are machine-scored</td>
<td>– Include both machine-scored and hand-scored questions</td>
</tr>
<tr>
<td>– Primarily assess Claim 1, but do include problem solving</td>
<td>– Primarily assess Claims 2, 3, and 4</td>
</tr>
</tbody>
</table>
Role of Teachers in the Development of Smarter Balanced Performance Tasks

- Collaborate to create Topic Maps
- Many authors of performance tasks are former teachers and professional educators
- Participate in the review process prior to publishing a performance task
- Provide constructive feedback following the Field Test
Performance Task Bundle (4–6 related PTs)

- Created by a team of teachers and professional educators
- Identifies real-world context
- Suggests CCSS
- Specifies grade level of performance task

Classroom Activity

- Authored by content specialist
- Extensive review process
- Aligned to content and mathematical practice standards

Introduces the context for the performance tasks to students
- Attempts to eliminate bias based on student experiences
General Guidelines for Development of Smarter Balanced Performance Tasks

• Integrate knowledge and skills across multiple content standards or strands

• Measure capacities such as depth of understanding, complex analysis, providing relevant evidence

• Reflect a real-world task and/or scenario

• Allow for multiple approaches

• Generally six questions within each performance task, which relate to a common Stimulus and build to an overarching question
Alignment for Each Question within a Performance Task

Each Question within A PT

- Standards for Mathematical Practices
- Depth of Knowledge
- Primary Claim (Claims 2, 3, or 4)
- Secondary Claim (Claim 1)

Primary Target (Includes a Content Domain)
Secondary Target (Includes a Content Domain)
CCSS (generally identifies two)
Each performance task consists of six questions, which provide evidence of the Claims as follows:

Between 0 and 2 questions within the performance task give students an opportunity to provide evidence of Claim 2.

Between 2 and 4 questions within the performance task give students an opportunity to provide evidence of Claim 3.

Between 1 and 2 questions within the performance task give students an opportunity to provide evidence of Claim 4.
# Anticipating Student Responses

In anticipating the student work, where will students **show success**?

| What parts of the task will students likely be successful? | In terms of knowing and doing mathematics, what does this indicate? (content and practices) |

In anticipating the student work, where will students **struggle**?

| What parts of the task will students likely be unsuccessful? | What understandings or skills might the students need to learn? (content and practices) |

Adapted from Silicon Valley Math Initiative (SVMI)
Claim 1: Concepts and Procedures

Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

• Generally DOK level 1
Claim 2: Problem Solving

Students can frame and solve a range of complex problems in pure and applied mathematics.

- Students can solve well-posed problems
- The components or information needed to solve the problem are contained within the stimulus for that specific question
- Generally DOK level 1 or 2
Claim 3: Communicating and Reasoning

Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

- Students communicate about the mathematics and problem solving
- Students justify solutions
- Students determine correct logic or arguments
- Students identify assumptions being used within the task
- Nearly always DOK level 3 and could include DOK levels 2 or 4
Claim 4: Data Analysis and Modeling

Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.

- Students solve problems in which all needed information is not contained within the stimulus for the specific question.
- Students make decisions regarding multiple solution pathways.
- Students analyze external information that may modify a previous solution.
- Students make improvements to a model or develop a new model based on a described situation or task.
- Generally DOK levels 2, 3, or 4
Four Principles of Performance Assessment

... targets skills and knowledge that matter
... is assessment for and as learning
... is learning by doing
... links curriculum, instruction, and assessment
Formatively Assessing What Students Are Learning

Teacher role:
• Elicit specific evidence about what is being learned
• Provide opportunities to surface insights and misconceptions
• Interpret and analyze the evidence
• Respond to the evidence with feedback and instruction

Student role:
• Use feedback to reflect on their own thinking, refine their work, and produce further evidence of learning
• Engage in self/peer assessment to reflect on and refine their thinking and work; this cultivates autonomy and awareness of their own learning

Performance Assessment

- Research Tasks
- Experiments
- Problem-Based Tasks
- Synthesis Tasks
- Extended Writing

Non-Performance Assessment

- Constructed Response
- Multiple Choice
- Selected Response
Role of Smarter Balanced Performance Tasks

- Provide measures of:
  - students’ achievement
  - academic growth
  - progress toward college and career readiness

- Involve significant interaction with stimulus materials and/or engagement in a real-world problem solution

- Integrate knowledge and skills across multiple content standards

- Allow for multiple approaches

- Include multiple items that are hand-scored

For more information, please read:

Smarter Balanced Performance Task Specifications
### Claims 2, 3, and 4: Verbs to Look For in Content Standards

<table>
<thead>
<tr>
<th>Claim 2: Problem Solving</th>
<th>Claim 3: Communicating &amp; Reasoning</th>
<th>Claim 4: Modeling &amp; Data Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understand</td>
<td>Understand</td>
<td>Model</td>
</tr>
<tr>
<td>Solve</td>
<td>Explain</td>
<td>Construct</td>
</tr>
<tr>
<td>Apply</td>
<td>Justify</td>
<td>Compare</td>
</tr>
<tr>
<td>Describe</td>
<td>Prove</td>
<td>Investigate</td>
</tr>
<tr>
<td>Illustrate</td>
<td>Derive</td>
<td>Represent</td>
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<tr>
<td>Analyze</td>
<td>Illustrate</td>
<td>Interpret</td>
</tr>
<tr>
<td></td>
<td>Analyze</td>
<td>Extend</td>
</tr>
</tbody>
</table>

- Understand
- Solve
- Apply
- Describe
- Illustrate
- Analyze

- Understand
- Explain
- Justify
- Prove
- Derive
- Illustrate
- Analyze

- Model
- Construct
- Compare
- Investigate
- Build
- Interpret
- Estimate
- Analyze
- Summarize
- Solve
- Represent
- Evaluate
- Extend
- Apply
Purposes of the Classroom Activity

- “Level the playing field”
- Background knowledge
- Vocabulary
- Eliminate bias
- Engage and motivate students
- Provide opportunity for collaboration and conversation
Relationship between Classroom Activity and Performance Task

- The Classroom Activity is NOT part of the assessment.
- Engagement, motivation, vocabulary, building background knowledge, and creating context are pivotal.
- Student performance may be directly affected (positively or negatively) by the quality of the instruction and scaffolds provided during the Classroom Activity—and throughout the year.
Accessibility and Support

• For the actual performance task, only the Universal Tools, Designated Supports, and Accommodations outlined in the Smarter Balanced guidelines can be provided.

• For the Classroom Activity, all students may be provided the same instructional supports, scaffolding, and/or accommodations used during regular instruction.
Let’s Unpack This Task

• Identify the math and anticipate issues
• Understand the Smarter Balanced Claims
• Alignment of the task
• Reflect on the purpose of Smarter Balanced performance tasks
Identifying the Mathematics and Anticipating Issues

• What do students need to know and be able to do to accomplish the task?

• What do you expect students to struggle with in this task?
Aligning the Task

• Which of the CCSS Standards for Mathematical Practice are engaged in this task?

• Which of the Smarter Balanced Claims are assessed by this task?
What Constitutes a Claim?

• Claims are broad statements of an assessment system’s learning outcomes.

• A claim is a statement of what students know and can do, based on the evidence they produce.

• Each Smarter Balanced Claim has multiple assessment targets—defined by content standards—to specify within the broader sense of the claim.
Overall Smarter Balanced Claims

Grades 3–8
Students can demonstrate progress toward college and career readiness in mathematics.

Grade 11
Students can demonstrate college and career readiness in mathematics.
## Math Claims

<table>
<thead>
<tr>
<th>Claim 1: Concepts and Procedures</th>
<th>• Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.</th>
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<tr>
<td>Claim 4: Data Analysis and Modeling</td>
<td>• Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.</td>
</tr>
</tbody>
</table>
Math Claims

Claim 1: Concepts and Procedures
Evidence of Claim 1 shows that students can “do math.”

Claim 2: Problem Solving
Evidence of Claims 2, 3, and 4 shows that students can apply mathematics to novel situations, think and reason mathematically, and use math to analyze empirical situations, understand situations better, and improve decisions.

Claim 3: Communicating Reasoning

Claim 4: Data Analysis and Modeling
Let's Do Math!

Performance Tasks
Putting it all together...
Sample Student Responses
Let’s look at how students handled this task.
Analyzing Student Work

Keep an eye out for:

• Common errors/misconceptions
• Successful approaches
• Examples of good explanations
Mathematical Flexibility
Procedural Flexibility and Strategic Thinking
QUESTIONS?
Much thanks for your presence, participation, level of engagement and your contributions today!

Ma Bernadette A. Salgarino, Ed.D.

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(408) 453 – 6974

http://tinyurl.com/wEdS4rdALT
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Language for Collaboration

**Requesting Ideas**
- What do you think makes sense?
- What should we write?
- Do you have an example?
- How else can we solve this problem?

**Suggesting Ideas**
- I think _____ would work well because__________.
- We could write ____________.
- What if we put __________since ____________.
- I think we should add ________________.

**Deciding on Ideas**
- Let us combine our ideas and write ________________.
- I think ____________ is the best example because ____________.
- Let us write ____________ capturing our ideas to ________.
- I would like to put ______________.

**Asking for Assistance**
- What does ______mean?
- Is there another way to say/solve ____________?
- Is this an appropriate ____________?
- How do you know if ______________ is correct?

**Restating Ideas**
- So, you said that ____________.
- So, you think that ____________.
- So, your idea is that ____________.
- So, your opinion is that ____________.

**Reporting Ideas**
- We thought of ____________.
- We came up with ____________.
- We determined that ____________ because ____________.
- We decided upon/ that ____________ because ____________.
- One idea/solution/method we had was ____________.

Adapted from: Kate Kinsella 2014
Daily Overview: http://www.oerv.eu/
Daily Overview: http://www.overv.eu/
Daily Overview: http://www.overn.eu/
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<table>
<thead>
<tr>
<th>Role Card #1</th>
<th>Role Card #2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Resource Manager</strong></td>
<td><strong>Facilitator</strong></td>
</tr>
<tr>
<td>§ Make sure that the team has all materials or resources for the tasks.</td>
<td>§ Make sure that everyone contributes and keeps the group mindful of each role to create productive collaboration.</td>
</tr>
<tr>
<td>§ Make sure your team cleans up by delegating tasks. You could say, “I will put away the _____ while you ________.”</td>
<td>§ Make sure you start the discussion in your group to maximize learning. You could say: “What do we think about the problem?” “Do we agree on all parts of the solution?”</td>
</tr>
<tr>
<td>§ Check for accuracy and clarity of thinking during discussion. May also check written work and keeps track of team point scores.</td>
<td>§ Help the team agree on an answer and lead discussion when there are disagreements.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Role Card #3</th>
<th>Role Card #4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Recorder/Reporter</strong></td>
<td><strong>Task Manager</strong></td>
</tr>
<tr>
<td>§ Make sure you keep notes on important thoughts expressed in the group.</td>
<td>§ Remind the team to stay on task and not to talk to students in other teams. You can suggest “Let us move on to the next step because ... “</td>
</tr>
<tr>
<td>§ Make final summary and lead the presentation while delegating other tasks needed for reporting results or your group work.</td>
<td>§ Listen for reasons and challenge your teammates to justify their thinking. “Why do you notice...” or “Why do you think that?”</td>
</tr>
<tr>
<td>§ Take notes for the team and notes should include phrases like, “For part one (or step one) ...” and explanations like, “Because part one (or step one) is ...”</td>
<td>§ Keep track of time and remind team how much time is left to complete assigned task.</td>
</tr>
</tbody>
</table>
Beyond Getting it Right! Fostering Mathematics Understanding in the Classroom

To view more student work, click the link below:
http://tinyurl.com/wEdS4rdALT