Preparing ALL K-2 Students to Make Sense of Word Problems

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Agenda

- Why Focus on Word Problems?
- First Grade Example
- Reading the Problem/Talk Moves/C.U.B
- Participation in Reading Comprehension
- Initial Problem Solving Strategies
- Word Problem Simulation
Context

- New CA State Standards (CCSS)
- Math Practices for students
  - Make sense of problems; persevere in solving them
  - Construct viable arguments; critique reasoning of others
- Goals for math conversations:
  - Students say second sentence
  - Students talk about each other’s thinking
  - Students write their mathematics
  - ELs produce language
  - ALL students participate
K/First Grade Example

There are 9 frogs in a pond. 5 frogs hop away. How many frogs are left in the pond?
Assess Language in Problem

• What language might be problematic for ELs? Why?
• What language might be problematic for struggling readers? Why?
• What could we do to assist students?
Reading the Problem

- First, teacher reads problem; follow with finger
- Then, call on “reader” to read problem
- Call on second student to read problem
- Last, call on emergent reader
Talk Moves

• **Revoicing**: repeat some/all of student response
• **Repeating**: student repeats what other said
• **Reasoning**: do you agree? Disagree? Why?
• **Adding On**: invite students to participate/clarify
• **Wait/Think Time**: you give it; students ask for it
• **Turn-and Talk**: students clarify/share
• **Revise**: students “revise” thinking as have new insights
C.B.U. Strategy (usually C.U.B)

- **Circle** the IMPORTANT numbers
  - Why is that number important in the problem?
  - What does that number tell us in the problem?
  - Is that number a “part”? The “whole”? 

- **Box** the action/comparison
  - Is action “putting together”? “taking apart”? 

- **Underline** the question
  - What are we trying to figure out?
Reading Comprehension/Participation Strategies/Talk Moves

- Prepare students for question/think time
- How does the story begin? *(Think time)*
- Whisper to shoulder partner how you think story begins *(Turn and Talk)*
- Call on student(s) to share
- What number is important at the beginning? Why is it important? What does it tell us?
- _____, can you repeat what _____ said about why the number is important? *(Repeating)*
- Let’s circle it *(C.B.U. strategy)*
Reading Comprehension/Participation Strategies/Talk Moves

- What happens next? *(repeat talk moves)*
- What ACTION is taking place? What are the frogs doing?
- Let’s BOX the action *(C.B.U. strategy)*
- Is this action “putting together” or “taking apart”? *(Think time; turn-and-talk)*
- What question is asked at the story end?
- Let’s UNDERLINE question *(C.B.U. strategy)*
Approaches to Problem Solving

- Story Mats
- Act it Out
Approaches to Problem Solving

- Model with Manipulatives
- Draw a Picture
- Write a Number Sentence

\[ 3 + \square = 5 \]
How Do Strategies Support ALL Students?

- All students participate and use language
- Repetition of language and reasoning
- All students practice “active” listening
- Kinesthetic/visual/auditory input
- Time for students to process/produce language
- Pictorial/number sentence connection to problem
Let’s Try It – 1st (2nd) Grade

There are 9 (29) cars in the parking lot. Some more cars entered the lot. Now there are 16 (51) cars. How many cars entered the lot?
Take Away Message

• Invest time in reading comprehension of story problems
• Use Talk Moves/C.B.U. Strategy
• Pose different “types” of problems
• Unknown in all positions
• Select numbers strategically to increase comprehension
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Presented by

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  – Students write their mathematics
  – ELs produce language
  – ALL students participate

K/First Grade Example

There are 9 frogs in a pond. 5 frogs hop away. How many frogs are left in the pond?

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• What language might be problematic for struggling readers? Why?
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C.B.U. Strategy (usually C.U.B)

- Circle the IMPORTANT numbers
  - Why is that number important in the problem?
  - What does that number tell us in the problem?
  - Is that number a “part”? The “whole”? (Think time; turn-and-talk)
- Box the action/comparison
  - Is action “putting together”? “taking apart”? (Think time; turn-and-talk)
- Underline the question
  - What are we trying to figure out?

Reading Comprehension/Participation Strategies/Talk Moves

- Prepare students for question/think time
- How does the story begin? (Think time)
- Whisper to shoulder partner how you think story begins (Turn and Talk)
- Call on student(s) to share
- What number is important at the beginning? Why is it important? What does it tell us?
- _____, can you repeat what _____ said about why the number is important? (Repeating)
- Let’s circle it (C.B.U. strategy)

Reading Comprehension/Participation Strategies/Talk Moves

- What happens next? (repeat talk moves)
- What ACTION is taking place? What are the frogs doing?
- Let’s BOX the action (C.B.U. strategy)
- Is this action “putting together” or “taking apart”? (Think time; turn-and-talk)
- What question is asked at the story end?
- Let’s UNDERLINE question (C.B.U. strategy)

Approaches to Problem Solving

- **Story Mats**
- **Act it Out**

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Common Core Standards For Mathematical Practice
Taken From CA Framework, published 2015

Kindergarten

### Focus, Coherence, and Rigor

When students represent addition and subtraction, this also supports mathematical practices as they use objects or pictures to represent quantities (K.OA.1\(▲\)), reason quantitatively to make sense of quantities and develop a clear representation of the problem (MP.2), mathematize a real-world situation (MP.4), and use tools appropriately to model the problem (MP.5). Math drawings also facilitate student reflection and discussion and help young students justify answers (MP.3).

### Table K-2. Standards for Mathematical Practice—Explanation and Examples for Kindergarten

<table>
<thead>
<tr>
<th>Standards for Mathematical Practice</th>
<th>Explanation and Examples</th>
</tr>
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<tbody>
<tr>
<td>MP.1 Make sense of problems and persevere in solving them.</td>
<td>In kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Real-life experiences should be used to support students’ ability to connect mathematics to the world. To help students connect the language of mathematics to everyday life, ask students questions such as “How many students are absent?” or have them gather enough blocks for the students at their table. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?”, or they may try another strategy.</td>
</tr>
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<td>MP.2 Reason abstractly and quantitatively.</td>
<td>Younger students begin to recognize that a number represents a specific quantity and connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. For example, a student may write the numeral 11 to represent an amount of objects counted, select the correct number card 17 to follow 16 on a calendar, or build two piles of counters to compare the numbers 5 and 8. In addition, kindergarten students begin to draw pictures, manipulate objects, or use diagrams or charts to express quantitative ideas. Students need to be encouraged to answer questions such as “How do you know?”—which reinforces their reasoning and understanding and helps student develop mathematical language.</td>
</tr>
<tr>
<td>MP.3 Construct viable arguments and critique the reasoning of others.</td>
<td>Younger students construct arguments using actions and concrete materials, such as objects, pictures, and drawings. They begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions such as “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking. They begin to develop the ability to reason and analyze situations as they consider questions such as “Are you sure that ________?” “Do you think that would happen all the time?” and “I wonder why ________?”</td>
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<tr>
<td>Standards for Mathematical Practice</td>
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<tr>
<td><strong>MP.4</strong> Model with mathematics.</td>
<td>In early grades, students begin to represent problem situations in multiple ways—by using numbers, objects, words, or mathematical language, acting out the situation, making a chart or list, drawing pictures, creating equations, and so forth. For example, a student may use cubes or tiles to show the different number pairs for 5, or place three objects on a 10-frame and then determine how many more are needed to “make a ten.” Students rely on manipulatives (or other visual and concrete representations) while solving tasks and record an answer with a drawing or equation.</td>
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<tr>
<td><strong>MP.5</strong> Use appropriate tools strategically.</td>
<td>Younger students begin to consider tools available to them when solving a mathematical problem and decide when certain tools might be helpful. For instance, Kindergartners may decide to use linking cubes to represent two quantities and then compare the two representations side by side, or later, make math drawings of the quantities. Students decide which tools may be helpful to use depending on the problem or task and explain why they use particular mathematical tools.</td>
</tr>
<tr>
<td><strong>MP.6</strong> Attend to precision.</td>
<td>Kindergarten students begin to develop precise communication skills, calculations, and measurements. Students describe their own actions, strategies, and reasoning using grade-level-appropriate vocabulary. Opportunities to work with pictorial representations and concrete objects can help students develop understanding and descriptive vocabulary. For example, students analyze and compare two- and three-dimensional shapes and sort objects based on appearance. While measuring objects iteratively (repeatedly), students check to make sure that there are no gaps or overlaps. During tasks involving number sense, students check their work to ensure the accuracy and reasonableness of solutions. Students should be encouraged to answer questions such as, “How do you know your answer is reasonable?”</td>
</tr>
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<td><strong>MP.7</strong> Look for and make use of structure.</td>
<td>Younger students begin to discern a pattern or structure in the number system. For instance, students recognize that $3 + 2 = 5$ and $2 + 3 = 5$. Students use counting strategies, such as counting on, counting all, or taking away, to build fluency with facts to 5. Students notice the written pattern in the “teen” numbers—that the numbers start with 1 (representing 1 ten) and end with the number of additional ones. Teachers might ask, “What do you notice when the number is _______?”</td>
</tr>
<tr>
<td><strong>MP.8</strong> Look for and express regularity in repeated reasoning.</td>
<td>In the early grades, students notice repetitive actions in counting, computations, and mathematical tasks. For example, the next number in a counting sequence is 1 more when counting by ones and 10 more when counting by tens (or 1 more group of 10). Students should be encouraged to answer questions such as, “What would happen if ________?” and “There are 8 crayons in the box. Some are red and some are blue. How many of each could there be?” Kindergarten students realize 8 crayons could include 4 of each color ($8 = 4 + 4$), 5 of one color and 3 of another ($8 = 5 + 3$), and so on. For each solution, students repeatedly engage in the process of finding two numbers to join together to equal 8.</td>
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**Operations and Algebraic Thinking**

K.OA

**Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.**

2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
### Standards for Mathematical Practice—Explanation and Examples for Grade One

<table>
<thead>
<tr>
<th>Standards for Mathematical Practice</th>
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| **MP.1**   
Make sense of problems and persevere in solving them. | In first grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or math drawings to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They are willing to try other approaches. |
| **MP.2**   
Reason abstractly and quantitatively. | Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. First-grade students make sense of quantities and relationships while solving tasks. They represent situations by decontextualizing tasks into numbers and symbols. For example, “There are 14 children on the playground, and some children go line up. If there are 8 children still playing, how many children lined up?” Students translate the problem into the situation equation \[14 - \_ = 8\], then into the related equation \[8 + \_ = 14\], and then solve the task. Students also contextualize situations during the problem-solving process. For example, students refer to the context of the task to determine they need to subtract 8 from 14, because the number of children in line is the total number less the 8 who are still playing. To reinforce students’ reasoning and understanding, teachers might ask, “How do you know?” or “What is the relationship of the quantities?” Students might also reason about ways to partition two-dimensional geometric figures into halves and fourths. |
| **MP.3**   
Construct viable arguments and critique the reasoning of others. | First-graders construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They practice mathematical communication skills as they participate in mathematical discussions involving questions such as “How did you get that?” or “Explain your thinking” and “Why is that true?” They explain their own thinking and listen to the explanations of others. For example, “There are 9 books on the shelf. If you put some more books on the shelf and there are now 15 books on the shelf, how many books did you put on the shelf?” Students might use a variety of strategies to solve the task and then share and discuss their problem-solving strategies with their classmates. |
**MP.4 Model with mathematics**

In the early grades, students experiment with representing problem situations in multiple ways, including writing numbers, using words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, or creating equations. Students need opportunities to connect the different representations and explain the connections. They should be able to use any of these representations as needed.

First-grade students model real-life mathematical situations with an equation and check to make sure equations accurately match the problem context. Students use concrete models and pictorial representations while solving tasks and also write an equation to model problem situations. For example, to solve the problem, “There are 11 bananas on the counter. If you eat 4 bananas, how many are left?”, students could write the equation $11 - 4 = 7$. Students should be encouraged to answer questions such as “What math drawing or diagram could you make and label to represent the problem?” or “What are some ways to represent the quantities?”

**MP.5 Use appropriate tools strategically.**

Students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when particular tools might be helpful. For instance, first-graders decide it might be best to use colored chips to model an addition problem.

Students use tools such as counters, place-value (base-ten) blocks, hundreds number boards, concrete geometric shapes (e.g., pattern blocks or three-dimensional solids), and virtual representations to support conceptual understanding and mathematical thinking. Students determine which tools are appropriate to use. For example, when solving $12 + 8 = \_\_\_\_$, students might explain why place-value blocks are appropriate to use to solve the problem. Students should be encouraged to answer questions such as “Why was it helpful to use ______?”

**MP.6 Attend to precision.**

As young children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning.

In grade one, students use precise communication, calculation, and measurement skills. Students are able to describe their solution strategies for mathematical tasks using grade-level-appropriate vocabulary, precise explanations, and mathematical reasoning. When students measure objects iteratively (repetitively), they check to make sure there are no gaps or overlaps. Students regularly check their work to ensure the accuracy and reasonableness of solutions.

**MP.7 Look for and make use of structure.**

First-grade students look for patterns and structures in the number system and other areas of mathematics. While solving addition problems, students begin to recognize the commutative property—for example, $7 + 4 = 11$, and $4 + 7 = 11$. While decomposing two-digit numbers, students realize that any two-digit number can be broken up into tens and ones (e.g., $35 = 30 + 5$, $76 = 70 + 6$). Grade-one students make use of structure when they work with subtraction as an unknown addend problem. For example, $13 - 7 = \_\_\_\_$ can be written as $7 + \_\_\_\_ = 13$ and be thought of as “How much more do I need to add to 7 to get to 13?”
MP.8
Look for and express regularity in repeated reasoning.

In the early grades, students notice repetitive actions in counting and computation. When children have multiple opportunities to add and subtract 10 and multiples of 10, they notice the pattern and gain a better understanding of place value. Students continually check their work by asking themselves, “Does this make sense?”

Grade-one students begin to look for regularity in problem structures when solving mathematical tasks. For example, students add three one-digit numbers by using strategies such as “make a ten” or doubles. Students recognize when and how to use strategies to solve similar problems. For example, when evaluating $8 + 7 + 2$, a student may say, “I know that 8 and 2 equals 10, then I add 7 to get to 17. It helps if I can make a ten out of two numbers when I start.” Students use repeated reasoning while solving a task with multiple correct answers—for example, the problem “There are 12 crayons in the box. Some are red and some are blue. How many of each color could there be?” For this particular problem, students use repeated reasoning to find pairs of numbers that add up to 12 (e.g., the 12 crayons could include 6 of each color [6 + 6 = 12], 7 of one color and 5 of another [7 + 5 = 12], and so on). Students should be encouraged to answer questions such as “What is happening in this situation?” or “What predictions or generalizations can this pattern support?”

Table 1-3. Methods Used for Solving Single-Digit Addition and Subtraction Problems

<table>
<thead>
<tr>
<th><strong>Level 1: Direct Modeling by Counting All or Taking Away</strong></th>
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<tbody>
<tr>
<td>Represent the situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend.</td>
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<tr>
<th><strong>Level 2: Counting On</strong></th>
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<tr>
<td>Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total, but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. The count is tracked and monitored in some way (e.g., with fingers, objects, mental images of objects, body motions, or other count words).</td>
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</tbody>
</table>

For addition, the count is stopped when the amount of the remaining addend has been counted. The last number word is the total. For subtraction, the count is stopped when the total occurs in the count. The tracking method indicates the difference (seen as the unknown addend).

<table>
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<tr>
<th><strong>Level 3: Converting to an Easier Equivalent Problem</strong></th>
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<tbody>
<tr>
<td>Decompose an addend and compose a part with another addend.</td>
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**Operations and Algebraic Thinking**

2.0A

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.1

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<td>In grade two, students realize that doing mathematics involves reasoning about and solving problems. Students explain to themselves the meaning of a problem and look for ways to solve it. They may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They make conjectures about the solution and plan out a problem-solving approach.</td>
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<td>MP.2 Reason abstractly and quantitatively.</td>
<td>Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. Students represent situations by decontextualizing tasks into numbers and symbols. For example, a task may be presented as follows: “There are 25 children in the cafeteria, and they are joined by 17 more children. How many students are in the cafeteria?” Students translate the situation into an equation (such as $25 + 17 = ___$) and then solve the problem. Students also contextualize situations during the problem-solving process. To reinforce students’ reasoning and understanding, teachers might ask, “How do you know?” or “What is the relationship of the quantities?”</td>
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<td>MP.3 Construct viable arguments and critique the reasoning of others.</td>
<td>Grade-two students may construct arguments using concrete referents, such as objects, pictures, math drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions such as “How did you get that?” “Explain your thinking,” and “Why is that true?” They not only explain their own thinking, but also listen to others’ explanations. They decide if the explanations make sense and ask appropriate questions. Students critique the strategies and reasoning of their classmates. For example, to solve $74 - 18$, students might use a variety of strategies and discuss and critique each other’s reasoning and strategies.</td>
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<td>MP.4 Model with mathematics.</td>
<td>In early grades, students experiment with representing problem situations in multiple ways, including writing numbers, using words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, or creating equations. Students need opportunities to connect the different representations and explain the connections.</td>
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Students model real-life mathematical situations with an equation and check to make sure that their equation accurately matches the problem context. They use concrete manipulatives or math drawings (or both) to explain the equation. They create an appropriate problem situation from an equation. For example, students create a story problem for the equation $43 + \Box = 82$, such as “There were 43 mini-balls in the machine. Tom poured in some more mini-balls. There are 82 mini-balls in the machine now. How many balls did Tom pour in?” Students should be encouraged to answer questions, such as “What math drawing or diagram could you make and label to represent the problem?” or “What are some ways to represent the quantities?”

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<th>MP.5</th>
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<td>In second grade, students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be better suited than others. For instance, grade-two students may decide to solve a problem by making a math drawing rather than writing an equation. Students may use tools such as snap cubes, place-value (base-ten) blocks, hundreds number boards, number lines, rulers, virtual manipulatives, diagrams, and concrete geometric shapes (e.g., pattern blocks, three-dimensional solids). Students understand which tools are the most appropriate to use. For example, while measuring the length of the hallway, students are able to explain why a yardstick is more appropriate to use than a ruler. Students should be encouraged to answer questions such as, “Why was it helpful to use ________?”</td>
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<td>As children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning. Students communicate clearly, using grade-level-appropriate vocabulary accurately and precise explanations and reasoning to explain their process and solutions. For example, when measuring an object, students carefully line up the tool correctly to get an accurate measurement. During tasks involving number sense, students consider if their answers are reasonable and check their work to ensure the accuracy of solutions.</td>
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<td>Grade-two students look for patterns and structures in the number system. For example, students notice number patterns within the tens place as they connect counting by tens to corresponding numbers on a hundreds chart. Students see structure in the base-ten number system as they understand that 10 ones equal a ten, and 10 tens equal a hundred. Teachers might ask, “What do you notice when ________?” or “How do you know if something is a pattern?” Students adopt mental math strategies based on patterns (making ten, fact families, doubles). They use structure to understand subtraction as an unknown addend problem (e.g., $50 - 33 = \Box$ can be written as $33 + \Box = 50$ and can be thought of as “How much more do I need to add to 33 to get to 50?”).</td>
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<th>MP.8</th>
<th>Look for and express regularity in repeated reasoning.</th>
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<td>Second-grade students notice repetitive actions in counting and computation (e.g., number patterns to count by tens or hundreds). Students continually check for the reasonableness of their solutions during and after completion of a task by asking themselves, “Does this make sense?” Students should be encouraged to answer questions—such as “What is happening in this situation?” or “What predictions or generalizations can this pattern support?”</td>
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## Addition and Subtraction Problem Types

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<tr>
<th>Join</th>
<th>Separate</th>
<th>Compare</th>
<th>Part-Part-Whole</th>
</tr>
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<tbody>
<tr>
<td><strong>Result Unknown</strong></td>
<td><strong>Result Unknown</strong></td>
<td><strong>Difference Unknown</strong></td>
<td><strong>Whole Unknown</strong></td>
</tr>
<tr>
<td>$5 + 7 = ?$</td>
<td>$12 - 7 = ?$</td>
<td>Keisha has 12 pencils. Juan has 5 pencils. How many more pencils does Keisha have than Juan?</td>
<td>Andrew has 5 green apples and 7 red apples. How many apples does he have?</td>
</tr>
<tr>
<td>Jeff has 5 toy cars. His friends gave him 7 more toy cars. How many toy cars does he have now?</td>
<td>Mark has 12 marbles. He lost 7. How many marbles does he have left?</td>
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<td></td>
</tr>
<tr>
<td><strong>Change Unknown</strong></td>
<td><strong>Change Unknown</strong></td>
<td><strong>Compare Quantity Unknown</strong></td>
<td><strong>Part Unknown</strong></td>
</tr>
<tr>
<td>$5 + ? = 12$</td>
<td>$12 - ? = 5$</td>
<td>Juan has 5 pencils. Keisha has 7 more pencils than Juan. How many pencils does Keisha have?</td>
<td>Andrew has 12 apples. 5 are green and the rest are red. How many red ones does he have?</td>
</tr>
<tr>
<td>Jeff has 5 toy cars. How many more cars does he need to collect to have 12 altogether?</td>
<td>Mark has 12 marbles. He lost some. Now he has 5 marbles. How many did he lose?</td>
<td></td>
<td></td>
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<tr>
<td><strong>Start Unknown</strong></td>
<td><strong>Start Unknown</strong></td>
<td><strong>Referent Unknown</strong></td>
<td></td>
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<tr>
<td>$? + 7 = 12$</td>
<td>$? - 7 = 5.$</td>
<td>Keisha has 12 pencils. She has 7 more pencils than Juan. How many pencils does Juan have?</td>
<td></td>
</tr>
<tr>
<td>Jeff has some toy cars. His friends gave him 7 more toy cars. Now he has 12 cars. How many did Ron start with?</td>
<td>Mark has some marbles. He lost seven. Now he has 5. How many marbles did he start with?</td>
<td></td>
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</tr>
</tbody>
</table>

Teacher Talk Moves

Revoicing-Clarifying
“So you are saying... Did I get that right?”

Repeating
“Who will repeat or rephrase what he said?”

Reasoning
“Do you agree or disagree with what was said, and why?”

Adding On
“What can you add to the idea she is building?”

Wait Time
“Take your time”

Revising
“Would anyone like to revise their answer?”
“I would like to revise my answer”
Talk Moves

Add On: More ideas, more details.

“I would like to add on to what _____ said”

Reasoning: To defend an idea.

“I agree because…”
“I disagree because…”
“That is true because…”

Repeating: To clarify what you heard someone say

“I heard you say…”
“_____ said…”
“Who can put this into their own words?”
“Who can restate what _____ said?”
“Who can repeat?”

Revoicing: To verify your thinking. To make sense of. To put into your own words.

“So you are saying…”
“What I think you said was…”
“Did you mean…?”

Think Time: To gather your ideas and thoughts.

“I would like some think time.”
“I would like to ponder on that question.”

Say More: To understand another’s thinking.

“Can you say more about that?”
“Tell us more about your thinking.”
“Can you expand on that?”
“Can you give us an example?”

Press for Reasoning: Explain reasoning.

“Why do you think that?”
“What is your evidence?”
“What convinced you that was the answer?”
“What makes you think that?”
“Why did you think that strategy would work?”
“How did you get that answer?”
“Can you prove that to us?”

Taken from: https://sites.google.com/site/mrmiranda103/talk-moves
**Math Constructive Conversation Skills Poster**

**Use Multiple Methods for Solving Ideas, & Understandings**

- **Prompt starters:**
  - What are we trying to do?
  - What is the problem asking?
  - How does the problem begin?
  - What happens?
  - What method is most useful? Why?
  - Which method is easiest to use?
  - Let’s try another way to solve this problem.
  - Let’s try a different way.
  - How can we represent this problem?
  - Let’s break it down.
  - How can we write the problem?
  - How can we draw or graph this?
  - How can we set up what we need to know?
  - How do we show what we mean?
  - What symbols can we use?
  - What do you think about this strategy for solving it?
  - What else could we do?
  - What are other ways to solve this?
  - What math rule are you using?
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